An Analysis of Chinese Junior High School Mathematics Textbooks: The Case of Quadratic Radical

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Abstract: This paper examines mathematics textbooks published by the People's Education Press (PEP) on the section of quadratic radical from the theoretical lens of Tyler's Rationale using two of the most relevant concepts (selecting learning experiences and elements to be organized). The analysis shows that the PEP textbooks demonstrate strong examples in aiming to cultivate students' logical thinking capacity. 'Addition' and 'reconstruction' are the main methods used to present maths history. The concise language style of the PEP textbooks can make it easy for students to memorize key mathematical concepts. However, less repeated information of quadratic radical may weaken the foundation for the association of knowledge for future learning. Furthermore, the PEP textbooks organize learning elements in the spiral way of rising based on three standards - continuity, sequence and integration. The spiral compilation and lack of connection between key knowledge points pose challenges to students and teachers and may lead to high mathematics expectations. The PEP textbooks emphasize the relationship between mathematical concepts and use the concept map to strengthen concept learning. More studies analyzing mathematics textbooks are needed to develop further an understanding of the concept map.

1. Introduction

In the past 20 years, there has been a growing interest in how students in different countries learn mathematics. As an important teaching resource, textbooks have attracted growing attention in educational research (Yuan, 2005; Zhao & Xu, 2018; Zhang, 2018; Zhang & Peng, 2015). According to the data of "Research on Mathematics Education" from 2013 to 2017 (Zhao & Xu, 2018), since the 21st century, the keyword comparative research has ranked No.14 in the statistical ranking. From the analysis of the domestic situation, the comparative study belongs to eight mainstream fields according to the data of the Journal of mathematics education from 2000 to 2016 (Zhang,2018). In a word, comparative study is a relatively indispensable subject.

From the results of international comparative studies, such as TIMSS (Trends of International Mathematics and Science Study and PISA (Program for International Student Assessment), the similarities and differences of different education systems in different countries are shown, which shows that students in East Asian countries have consistently performed well (Dae et.al,2014; Zhang

& Peng, 2015; Zhang & Shen,2017). The performance of students' mathematical literacy can be influenced by various factors, for instance, textbooks (Fan & Zhu,2007; Zhu & Fan, 2006) and classroom video. Research has also suggested that curriculum reform with the consideration of the local culture is one of the important factors for the success of PISA in Shanghai (Zhang & Peng, 2015; Tan, 2012, 2013; Sellar & Lingard, 2013).

Since textbook is one of the key factors that account in the teaching process and predominantly guide what teachers teach and students learn inside the classroom, the analysis of textbooks may provide insights into possible causes of students' performance differences. In particular, this paper explores possible factors within textbooks that may affect the differences of Chinese junior high school students' mathematics achievements from the perspective of teaching quadratic radical.

The content of quadratic root can be difficult and complex for junior high school students, which directly affects students' learning of high school mathematics. There are many concepts in the content of quadratic radical, such as many invisible conditions and strong logic. The content of the quadratic radical has high requirements on the operation ability, which is the first inverse operation that students face after four operations. The teaching materials of junior and senior high school all involve quadratic radical. Quadratic radical is an important part of number and algebra. It runs through the study of mathematics in junior high school and is one of the turning points and obstacles in the process of learning from primary school to junior high school. On the one hand, similar to negative numbers, radical involves irrational numbers, even imaginary numbers, which is the expansion of students' original number system. On the other hand, radical also involves operation, a new form of operation, that is, in addition to the operation of the radical itself, the inverse operation of the nth power of numbers. In junior high school, the extension of radical can involve quite a lot of content, Pythagorean theorem, trigonometric function, fractional index and so on.In such contexts, this paper explores the content of quadratic radical in the PEP textbooks in relation to Tyler’s Rationale, and critically discusses the textbooks from the perspectives of learning experiences and elements to be organised, aiming to provide new understanding and insights. This paper will first consider the concept of selecting learning experiences, along with the four kinds of learning experiences. This is followed by explanation, citing examples from textbooks and conclusion or suggestions. After that, the concept of elements to be organized will be analyzed, together with the three standards of effective organizational learning elements. In the end, the conclusion offers a deeper understanding of the PEP textbooks and suggests possible directions for future research.

2. Literature Review

Following the implementation of the mathematics curriculum standard for full-time compulsory education (MoE, 2011), mathematics textbooks in China have been adjusted and modified accordingly, and scholars have expressed their views and pointed out possible problems (Zhang & Peng, 2015; Kong, 2007; Zheng, 2000; Wang, 2013). A key issue is within the ‘spiral way of rising’ structure of knowledge in textbooks (Li, 2010; Ye, 2017; Zhou, 2019; Dong, 2012; Kong, 2006). Among the three types of compilation of teaching materials in middle school maths textbooks - linear, spiral and circular, the ‘spiral way of rising’ structure arranges teaching materials from simple and basic to complex and advanced in a way where important concepts and thinking methods appear repeatedly and gradually (ibid.). The concept of "spiral curriculum" was put forward by Bruner in the 1960s, referring to the curriculum and textbook design that promotes the development of students' cognitive ability based on the "conceptual structure" of the subject knowledge. This "spiral way of rising" structure in curriculum design and textbook compilation has been widely used in China in the past decades (Zheng, 2000; Wang, 2013; Kong, 2017; Li, 2012; Ye, 2010). However, scholars have argued that the spiral interval can be too long for students to make a connection between the new

content and the knowledge learnt (Kong, 2007; Ye, 2017; Li, 2010). This may lead to less systematic and coherent arrangement and delivery of the overall body of knowledge, which in turn undermines the effectiveness of teaching students independent and critical thinking, curiosity, rational generalisation, abstract thinking, and scientific spirit (Jiang, 2015). The lack of connections between different points of knowledge across different learning stages thus poses challenges to the effectiveness of the 'spiral way of rising' structure in textbooks.

In addition, the various purposes and degree of application of mathematical history in textbooks may also have crucial effects on students' learning (Shen, 2018; Fan et al, 2019; Wang & Hu, 2019). At present, some contents of mathematics history have been incorporated into the textbooks of junior middle school mathematics textbooks in China. For example, Al-Khwarizmi developed a formula for systematically solving quadratic equations and using Pythagoras's story to explain Pythagorean Theorem. However, studies have pointed out some shortcomings and problems about the ways history of mathematics is incorporated in textbooks (Shen, 2018; Fan et al, 2019; Wang & Hu, 2019; Liu,2013; Jiang, 2016; Song, 2012). The content of mathematics history in the PEP textbooks is limited, single-arrangement, and mostly presented in the tone of adults, which is more abstract and general (Liu, 2013; Fan et.al, 2019). Content in mathematical history remains to be further enriched. The content application level also needs to be improved (Wang & Hu, 2019; Liu, 2013; Jiang, 2016; Song, 2012). At the moment, the PEP textbooks are more focused on guiding students to reproduce the mathematical problem-solving process and to explain the history of mathematics in-depth (Fan et al, 2019; Wang & Hu, 2019; Liu, 2013). These would require students to have a high level of critical thinking skills, which may be too challenging for many students. Based on Tzanakis and Arcavi's research (2002), Wang (2012) defines five ways to use mathematics history in mathematics teaching materials (embellishment, addition, replication, adaptation and reconstruction). The adaptation way of mathematics history refers to changing the question of mathematics history in order to adapt to the teaching situation setting. There is a limited proportion of adaptation way of mathematics history materials in the PEP textbooks (Wang & Hu, 2019), which might need to increase.

Research on the teaching and learning of quadratic radical has remained a key focus in the academic debate. Students often experienced difficulties in dealing with complex quantitative relations, concepts and setting of contents, due to the abstract nature of mathematics (Li, 2019; Ou, 2019). Frederick (2005) shows that Students in East Asian countries talk less in class, but they are exposed to more teaching content. One concern is that if classroom teaching is dominated by students' self-questioning and self-reflection, it may lead to lower student exam grades compared to traditional teacher dominated classroom teaching mode (Dun et al,2018). As a result, Chinese textbooks are often not designed or used to encourage group discussion or self-question inside the classroom, but rather to facilitate teacher dominated teaching. Nevertheless, researches show that meaningful learning can still take place in teacher dominated classrooms (Fredrick, 2005). East Asian classrooms are often teacher dominated. This seems to be at odds with the recent trend towards student-centered classrooms (Dun et al, 2018; Wang, 2019; Gao & Sheng, 2014). Yet a well-structured teacher-led lesson may be able to assist students in constructing their mathematics knowledge more effectively. Teacher dominance does not necessarily lead to passive, receptive learning. Much depends on whether teachers' lesson delivery can stimulate students to become actively engaged in mathematics learning (Fredrick, 2005; Hiebert et al, 2003; Mullis et al, 2000; Prawat, 1991). In this regard, Tyler (1949) similarly argues that the key is whether students can be stimulated to participate in learning.

Tyler's (1949) seminal work on curriculum and instruction offers a system of general procedures and principles of curriculum and teaching as well as the objective model of curriculum editing. Tyler "attempts to explain a rationale for viewing, analyzing and interpreting the curriculum and instructional program of an educational institution" (Tyler, 1949, p. 143). To analyse the PEP maths textbooks, this paper draws from the two most relevant concepts (selecting learning experiences,

elements to be organized) from the two chapters. To further explain, first, learning experience refers to "the interaction between the learner and the external conditions in the environment to which he/she can react" (Tyler, 1949). Tyler illustrated four characteristics of learning experiences useful in attaining various types of objectives (learning experiences to develop skill in thinking, learning experiences helpful in acquiring information, learning experience helpful in developing social attitudes, learning experience helpful in developing interests). In the paper, the textbooks are analyzed based on the four characteristics of learning experience. Second, elements to be organized refers to "the elements of that curriculum which serve as the organizing threads" (Tyler, 1949, p. 224). In maths teaching, the organizing elements often mean maths concepts and skills. Thus, the first key step of organizing learning elements is to identify the basic concepts of major importance in maths learning. All of whom conceived of curriculum not as a product, such as a curriculum guide, but as the experience students have in educational settings (Wraga, 2017). Thus, the elements to be organized in educational settings can be regarded as the structure of the curriculum.

Aiming to enhance understanding and improve teaching materials, this paper explores the elements to be organized and selecting learning experiences of Chinese maths textbooks through the theoretical lens of Tyler's Rationale. Particularly, considering that few studies on textbook analysis are based on Tyler's Rationale, and that among them, most focus on the steps of clarifying educational purposes or the evaluation, this paper contributes to a much-needed debate on what distinctive aspects of PEP textbooks may particularly benefit students' learning. The analysis aims to provide insights into reasons for student's mathematics achievement, offering implications for improving curriculum organization. Based on Tyler's Rationale, the following section will analyze the PEP maths textbooks from the perspectives of learning experiences and elements to be organized.

3. Analysis and Discussion

3.1.The Analysis of Learning Experience

According to Tyler (1949, p. 121) "learning experience" refers to the interaction between the learner and the external conditions in the environment that make him/her respond. In his rationale Tyler conceived of education as experience (Wraga, 2017). Learning takes place through the active behavior of the learner. Learning much depends on the interaction between the learner and the external conditions in which he/she can react, not what the teacher does. Learning experiences can achieve the same goal, and the same learning experience can also produce different results. Among the many versions of junior high school mathematics textbooks in China, the version from the People's Education Press (PEP) has the widest scope of application and the largest number of users (Li, 2019). Thus, the following section analyzes the PEP maths textbooks from four perspectives of learning experiences.

3.1.1. Learning Experiences to Develop Skills in Thinking

Tyler (1949, p. 206) categorizes "thinking" into three kinds: inductive thinking, deductive thinking, and logical thinking. The section on quadratic radical in the PEP maths textbooks clearly reflects how these thinking skills are designed to be taught and enhanced. First, inductive thinking involves drawing generalizations from several items of specific data (Tyler, 1949, p. 206). For example, in PEP Grade 8 Volume 2 (p. 12), addition and subtraction of quadratic radical pointed that after showing the calculation process of $\sqrt{8}+\sqrt{18}$, the arithmetic of addition and subtraction of quadratic radical is summed up. This reflects the inductive thinking as it requires students to induct the given example of computing. Second, deductive thinking involves applying one or more generalizations to specific cases (Tyler, 1949, p. 206). Deductive thinking appears repeatedly in the example exercises

in the textbook. After learning the concept, it is deduced in the example exercises. For example, after the stem, the exercise of calculating the following formula is a case of deductive thinking, as they are just the deduce of the same principle: (1)$\sqrt{36}$; (2) $-\sqrt{0.81}$; (3) $\pm\sqrt{\frac{49}{9}}$ (PEP Grade 7 Volume 2 (p. 46). Third, logical thinking involves the arrangement of assumptions, premises, and conclusions in a way to develop a logical argument (Tyler, 1949, p. 206). Logical thinking is clearly reflected in the "exploration" and "mathematical activities" sections of PEP. For example, in PEP Grade 7 Volume 2 (p. 59), since the ones digit of 59219 is 9, can you define the ones digit of $\sqrt[3]{59319}$? This problem cannot be solved by inducing or deducing. The stem has the premise that Hua Luogeng, the best-known Chinese mathematician, figured out the intelligence questions in the nearby-seat passenger's magazine. The logical thinking refers to that there is no direct correspondence between the questions in the background, the way of solving problems or the methods and the steps of solving problems with the examples and examples in the textbooks. The questions are open, or the methods of solving problems are unconventional, or there are no fixed solutions or no unique results. For this example, on page 59, since the methods of the problem is unconventional, it is an example of logical and deductive thinking following the premise of Hua Luogeng. In short, the three kinds of thinking have their own characteristics, and a learning topic can cultivate 2 or 3 kinds of thinking at the same time.

3.1.2. Learning Experiences Helpful in Acquiring Information

The PEP textbooks can help students significantly improve their thinking ability, especially their inquiry ability. The mathematical problems in teaching materials are more expressed in mathematical language and symbols, and they are mostly set in an abstract context unrelated to the real world. Multiple steps are involved in solving maths problems, and the proof process is often complex and multi-leveled (Charalambous, 2010; Dae, 2013; Michoi, 2013). This is inarguably conducive to the cultivation of thinking skills. Tyler (1949, p. 209) proposed suggestions to "overcome defects in the acquisition of information, for instance, to select only important information to include as worthy of remembering and using this important information frequently and in varied contexts". In this section, the PEP textbooks are researched from the two points, to select only important information and frequently using important information.

The first part is about selecting only important information to include as worthy of remembering. The PEP maths textbooks have a concise, simple and generous language style. They only select and include information that is deemed necessary, and there is no long speech. For example, in PEP Grade 8 Volume 2 (p. 6), the textbook only showed the product rule of quadratic radical without including other aspects such as context and explanation, which can be considered as less important or too many words. The short and concise contents display the language in a way that benefits learners to remember and understand (He et.al, 2014).

The second part is using these important items of information frequently and in varied contexts. PEP's knowledge points are presented in chapters, but each chapter is relatively independent. In the two volumes of textbooks, the "quadratic radical" is embodied in two chapters (Grade 7 Volume 2, p. 40-62; Grade 8 Volume 2, p. 1-19). The quadratic radical is nor frequently appears in other contents. In different situations, if the root form can appear many times, it can enhance the possibility of association in the future, and also give the processed information more meaning. The emergence of information is not only for memory, but also for providing more solutions to problems (Anne & Mason, 2005). Thus, the frequent mention of quadratic radical in different chapters can lead to the possibility of association in the future.

In short, the concise language style in the PEP textbooks is helpful to students' memory. Less repeated information of quadratic radical in PEP will weaken the possibility of association in the future.

3.1.3. Learning Experiences Helpful in Developing Social Attitudes

Objectives that can be classified as developing social attitudes include those emphasized in social studies, literature, arts, physical education, and extracurricular activities. Attitudes are defined as a tendency to react even though the reaction does not actually take place (Tyler, 1949, p. 212). To cultivate the learning experience of social attitude does not mean depriving the students of their feelings or force them to change their attitude. Learners will only change their social attitude through their new understanding of the environment or their satisfaction in learning. In the setting of mathematical history, the PEP textbooks tend to use addition way and reconstruction way of mathematical history. For example, in the section 'reading and thinking' of PEP Grade 7 Volume 2 (p58), the example of mathematical history of Pythagoras is showed with the content, to prove that $\sqrt{2}$ is an irrational number. Assumed that there are two prime numbers p, q, and $\sqrt{2}$is a rational number, to make $\sqrt{2} = \frac{p}{q}$. Finally, coming to the conclusion that $q^2 = 2s^2$, which contradict that p, q are prime numbers. Thus, $\sqrt{2}$ is an irrational number. The PEP textbooks focus on exploration and explain the history of mathematics in-depth instead of the origin and development of mathematics. Thus, it is a case of the reconstruction way of mathematics history. In the PEP textbooks, the contents of the mathematical history only come at the end of every chapter. They are thus more likely to be ignored, and their presentation style of addition and embellishment may not play any significant role in cultivating students' social attitudes (Wang, 2012; Liu, 2013).

The PEP version of teaching mathematics history is more about guiding students to use mathematicians' methods to reproduce the problem-solving process (Wang & Hu, 2019; Song, 2012; Song, 2018). Yet the in-depth explanation of the history of mathematics knowledge points and the higher requirements for students' thinking skills in the textbooks may also weaken students' interest in and motivation for learning. Thus, in order to help students, cultivate their attitude towards maths and improve their interests in learning maths, promoting the transfer of knowledge and skills may be key. In the process of solving mathematical problems, students can apply the new learning knowledge to the problems of different situational settings, which can strengthen the concept and facilitate the knowledge transfer.

In summary, "reading and thinking" and "mathematical activities" can be compiled into adaptation way of maths history to inspire students. They can also be directly moved to the middle part of the chapters to increase students' multi-situational setting training, which is conducive to students' knowledge transfer (Anne & John, 2005).

3.1.4. Learning Experience Helpful in Developing Interests

Interests are often emphasized as important educational objectives, because what students are interested in largely determines what they attend to and frequently what they do (Tyler, 1949, p. 216). Studies have pointed out that mathematics interest and mathematics expectation influence each other and are positively related to mathematics achievement (Wang et.al, 2019). Mathematics interest reflects students' willingness and enjoyment through learning mathematics.

Thus, making textbooks more interesting, vivid, and engaging may help to improve students' learning interest. In this regard, more visually beautiful pictures, illustrations, colors, and typesetting may help. For example, in PEP Grade 8 Volume 2 (p. 29), there is a water pool problem. The surface of the water is a square with a side length of 10 feet. In the middle of the pool, there is a reed, which

is one foot higher than the water surface. If you pull this reed to the middle of the pool side, its top just reaches the water surface by the pool side. What are the depth of the water and the length of the reed? On page 29 of PEP 8, the movement of reeds in daily life is selected in question 10. The unit of length in ancient China "Chi" is taken as the unit of measurement. The swing of plant reed in nature is a common natural phenomenon. This helps to connect the maths problems to real-life scenarios, which is conducive for students to better understand the practicality of learning maths. For another example, in PEP Grade 7 Volume 2 (p. 42), Lily wants to use a piece of square paper with an area of $400\,cm^2$ to cut out a rectangular piece of paper with an area of $300\,cm^2$. The title gives the names of the characters, the life background of the paper-cut, establishes the quantitative relationship, and solves the root formula to calculate the result. The problem is contextualized in real-life scenarios. In the process of solving the problem, students may have a higher level of learning satisfaction because the problem seems to be fun and real. Teaching materials as such may thus improve students' learning motivation for quadratic radical. Mathematics interest has a direct influence on students' mathematics achievement (Wang & Ma, 2019).

In short, it is helpful to contextualise the teaching of "quadratic radical" in the scenes of daily life as much as possible so that the concept can be connected with the actual things in life. This way, students can also vividly experience the practicality of mathematics and gaining more satisfaction from solving problems. After the discussion of learning experience, as the order of Tyler's Rationale, the key step is the definition of elements that need to be organized, which will be explored in the following section.

3.2. The Analysis of Elements that Need to be Organized

The nine-year compulsory education junior high school mathematics PEP textbook is the most widely used edition in various regions of China, adopted by schools in provinces such as Fujian, Yunnan and Heilongjiang. As a key part of the scheduled curriculum, textbooks reflect the educational philosophy and teaching values to a certain extent (Zhu & Fan,2007). Thus, the following analysis aims to offer insights into elements to be organized of quadratic root in this regard, along with the three standards of effective organizational learning elements. Furthermore, a brief summary will be presented.

Tyler (1949, p. 224) noted that "the elements could serve as an organizing element in achieving continuity and sequence". Before the implementation of teaching, mathematics teachers need to make clear the basic concepts of organizational elements, and help students understand these learning elements in teaching practice. After mastering the concept, learners are trained with the corresponding skills to ensure the organization of learning elements. For example, when the concept of "quadratic radical" appears in junior high school, in PEP Grade 7 Volume 2 the concepts of square root, arithmetic square root, power, square root, cube root, and irrational number are first presented at basic levels. Then in PEP Grade 8 Volume 2, the contents progress to more complicated learning elements such as the four basic operations of quadratic radical, simplification, rationalization, operation skills, extension to fractional index, root equation, plane vector, and trigonometric function. The content of quadratic radical is presented repeatedly and gradually in the spiral way of rising (Ye, 2017; Zhou, 2019; Dong, 2012; Kong, 2016; Kong, 2017). The first mention of quadratic radical is in Grade 7 Volume 2. After a year, without much connection, the content of quadratic radical appears for the second time. The lack of coherent connection may make it difficult for students to recall the information and weaken the possibility of association afterwards (Kong, 2007; Ye, 2017; Li, 2010).

Consequently, teachers often experience difficulties in grasping the connection between different parts (Kong, 2007). Unsurprisingly, students may also find it challenging to correctly use the new solutions, instead of the previously learnt solutions, to solve maths problems. Zheng (2000) suggests that the 'spiral way of rising' structure may be a mismatch with the mentality and thinking pattern of

teachers and students for complete, thorough knowledge teaching and learning in a "once and for all" fashion.

The intervals of the ‘spiral way of rising’ structure in textbooks may disrupt the systematization of knowledge. Studies have shown that the spiral intervals of the knowledge structure appear in PEP textbooks, which could weaken the integrity and systematization of knowledge (Wang, 2013; Zheng, 2000; Li, 2020; Men, 2016). However, some (Men, 2016; Hess et al.,1987; Wu et al, 2013) suggest that the PEP textbooks contain more and advanced knowledge, which reflects the systematization of knowledge in a different way. That is also one of the typical characters of Chinese mathematics textbooks. Scholars have argued that more advanced content is covered in the East Asian textbooks, suggesting that teachers and students in East Asia have high expectations of students in terms of the level of mathematics they are able to learn (Hess et al., 1987; Stevenson, 1987; Wang & Ma et al, 2019). Mathematics expectation, which is related to mathematics self-confidence, may be contributed to higher students’ mathematics achievement (Ma et al, 2019).

Thus, the spiral compilation and lack of connection pose challenges to students and teachers. However, the challenges may lead to high mathematics expectation, which can be conducive to students’ mathematics achievement (Ma et. al., 2019).

3.3. Continuity, Sequence and Integration

Tyler (1949, p. 233) put forward three standards of effective organizational learning elements: continuity, sequence and integration. Continuity is the arrangement and superposition of curriculum elements. PEP Textbooks focus on the secondary radical content in chapters, with less overlap. The PEP textbooks provide more new concepts and focus on them early on. For example, the content about quadratic root in the PEP textbooks first appears in the second chapter of the second volume of the seventh-grade textbooks. The chapter first teaches the square root, arithmetic square root, the number of the square root, the multiplier and the square root, and the range of the value of the quadratic root. After that, it explains the cube root, including the definition of cube root, open cube and cubic, root index, irrational number and real number, and the approximate range of estimating irrational number. The second volume of the eighth-grade textbook also mentioned the relevant content, including the meaning of the square root of arithmetic, algebra, addition and subtraction of the quadratic radical, multiplication and division, the simplest quadratic radical and so on. For example, in PEP Grade 7 Volume 2 (p. 53), an infinite acyclic decimal is called an irrational number, such as $\sqrt{2}, -\sqrt{5}, \sqrt[3]{3}$,which are all irrational numbers. π =3.14159265...is also an irrational number. In page 53, the concept of irrational number is introduced in an abstract way. First of all, the rational number is defined. Any finite decimal or infinite recurring decimal is a rational number. On the contrary, infinite non-cyclic decimal is opposite. On the negation of the definition of rational number, the concept of irrational number is abstractly defined. Thus, the concept of irrational number is based on the definition of rational number, which reflects the continuity of the PEP textbooks.

Sequence refers to the establishment of more in-depth and extensive learning experience in the original experience. PEP textbooks are mainly put quadratic root in two chapters. Seventh grade focuses on the concept of quadratic radical and eighth grade focuses on the operation of quadratic radical. For example, in PEP Grade 7 Volume 2 (p. 58), a number is known to be 0.1010010001000100001. By looking from left to right, there is one more number between adjacent ones. Is this number rational and why? This question examines the concept of irrational numbers. When we look at the known numbers from left to right, there is one more 0 in the two ones, which shows that the known numbers are regular numbers. Known numbers are infinite decimals, but they are not cyclic decimals. Therefore, known numbers are infinite non-cyclic decimals and belong to irrational numbers. This exercise puts forward higher requirements for mastering the concept of

irrational number, which helps to cultivate students' thinking. At the same time, it also investigates students' language expression, which is a case of sequence.

Integration refers to the horizontal connection between learning experiences and the unity of students' views, skills and attitudes. The PEP textbooks emphasize the systematization and hierarchy of mathematical knowledge, and the connection between concepts. It is helpful to use the schema of the concept map to strengthen concept learning. Cognitive psychology research shows that people need organized schemata to understand new information, and concepts are stored in the brain in the form of schema (Bao, 2008). For example, in PEP Grade 7 Volume 2 (p. 57), the rational numbers and irrational numbers can be called as real number, and the classification is as the following graph. When explaining the concepts of rational number and irrational number, we use the intuitive presentation method and list the graphs to show the subordination relationship among real numbers, rational numbers and irrational numbers. Rational numbers and irrational numbers are divided by positive and negative respectively. Rational numbers include 0, the definition of rational numbers and irrational numbers. In the learning of irrational number and the expansion of students' number system, the relationship between the knowledge of rational number and the concept of irrational number should be constructed to deepen the understanding of the concept of irrational number. The presentation of graphs and charts, the concept of irrational number mainly constructs the connection between the old and the new knowledge, so that students can accept the concept of irrational number, the structure between the number system, the types of rational number and irrational number. It thus reflects the integration.

In short, from the aspect of quadratic radical, the PEP textbooks followed the three standards of effective organization, namely, continuity, sequence and integration. The PEP textbooks arrange the contents of the second root form according to chapters, with less overlap. PEP will provide more new and advanced concepts. The content of the PEP textbooks has less overlap and less connection between different knowledge points. However, the PEP textbooks emphasize the relationship between mathematical concepts and use the schema of the concept map to strengthen concept learning.

4. Conclusion

To sum up, this paper has analyzed the PEP textbooks on the section of quadratic radical from the theoretical lens of Tyler’s Rationale, using two concepts, which are selecting learning experience and elements to be organized.

This paper first considers the concept of selecting learning experiences, along with the four kinds of learning experiences. The PEP textbooks demonstrate strong examples in aiming to cultivate students’ logical thinking capacity and problem-solving skills. The maths problems used in the PEP textbooks tend to be multistep, displaying mostly in straight-forward mathematical language and focusing on the use of symbols. The concise language style in the PEP textbooks can make it easy for students to memorize maths concepts, rules, and functions. However, less repeated information of quadratic radical in the PEP textbooks may weaken the foundation for the association of knowledge for future learning. In the PEP textbooks, the contents of the mathematical history only come at the end of every chapter. They are thus more likely to be ignored, and their presentation style of addition and embellishment may not play any significant role in cultivating students’ social attitudes (Wang,2012; Liu,2013). So, "reading and thinking" and "mathematical activities" can be compiled into the adaptation way of maths history to inspire students. They can also be directly moved to the middle part of the chapters to increase students' multi-situational setting training, which is conducive to students' knowledge transfer (Anne & John, 2005). Putting the exercises in the scenes of the daily life can help to make students gain more satisfaction from solving maths problems.

After that, the concept of elements to be organized be analyzed, together with the three standards of effective organizational learning elements. PEP textbooks organize elements in the spiral way of rising with three standards, continuity, sequence and integration. The spiral compilation and lack of connection pose challenges to students and teachers. However, the challenges may lead to high mathematics expectation, which can be conducive to students' mathematics achievements (Ma et al, 2019). The PEP textbooks arrange the contents of the second root form according to chapters with less overlap and less connection between different knowledge points but more independent new concepts.

However, the PEP textbooks emphasize the relationship between mathematical concepts and uses the schema of the concept map to strengthen concept learning. It thus might be meaningful to investigate further the functions of the concept map. The content of quadratic radical in PEP contains multiple concepts. Setting up a concept map can form a good cognitive structure in the process of students mastering the concept (Bao, 2008). The structure of schema may help students memorize knowledge and related concepts, as well as the relationship between concepts. Further studies are thus needed to offer a deeper understanding in this regard to shed light on links between textbooks and students' mathematical achievements.

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